On the Value of Velocity Tuning©

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While sorting out accuracy problems with a varmint-class benchrest rifle, this question arose: How much can you fix with tune? In a properly-functioning, well-made conventional 6 ppc varmint class benchrest rifle using equally good, high quality and appropriate components, equal and appropriate loading technique, rests, etc., how much, on the average, can velocity tuning affect the size of a 100 yard, 5-shot group? How much bigger than a group produced by rounds charged at the "sweet spot" would a group be if fired with loads charged at a nearby "sour spot?"

Upon asking a number of experts – hall-of-fame members and other serious students of internal ballistics as applied to the benchrest game – a consensus was not formed. Several experts, however, suggested re-reading chapter 4 in Harold R. Vaughn's "Rifle Accuracy Facts," commenting that the answer would surely lie within. Pursuant thereto, is the following exploration of Vaughn's published model of velocity tuning. The Vaughn model was used to answer, at least in part, the aforementioned question.

Before continuing, a couple of clarifications are offered. Here, "tuning" means "velocity tuning" adjusting velocity by changing the weight of the charge, leaving all other controllable factors unchanged insofar as is possible. Furthermore, the model explains, and this study investigates, only one factor that affects only the vertical dimension of a group.

Vaughn's Sine Wave

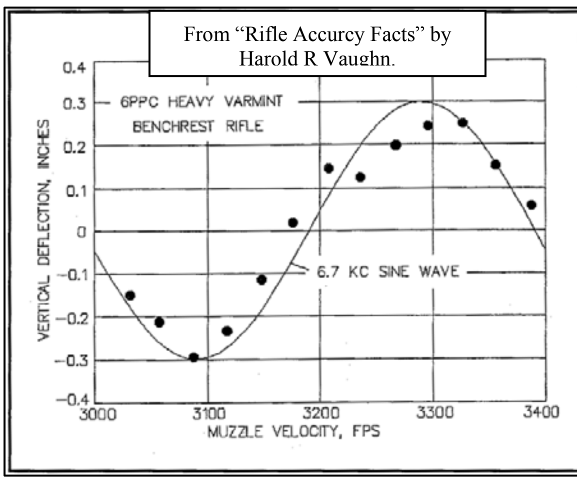
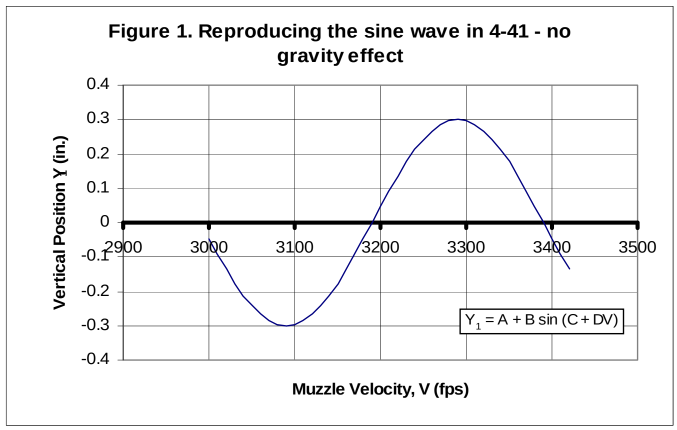


Figure 4-41 - Plot of the vertical positions of group centers for various muzzle velocities on a 6PPC benchrest rifle. The effect of varying gravity drop due to differences in velocity has been removed.

Vaughn presents, in figure 4-41 of "Rifle Accuracy Facts," a plot of corrected vertical deflection of the point of impact as a function of muzzle velocity for a particular HV benchrest rifle. The data plotted are points of impact of groups fired at different muzzle velocities; the value adjusted to remove the effect of gravity. A curve on the same scale that appears to be a sine wave is fit to the data points. The accompanying text states, or at least indicates, that the plotted vertical deflection is the displacement of the point of impact at 100 yards caused solely by the angular and linear motion of the muzzle plane of the barrel. In the following, the wave is treated as a sine wave. This is, at least, a useful simplification, although the period probably compresses at lower velocities.

The Vaughn sine wave is not a plot of barrel position or muzzle angle as a function of time; the sort of thing that would result from analysis or measurement of a barrel's motion caused by an impulse or other stimulus. Vaughn's sine wave is, however, related directly to such a motion. Vaughn's book does not describe the analysis relating natural frequencies to the point of impact sine wave, although the relationship appears to be part of the underlying work. He notes that the sine wave is related to a 6700 Hz response of the rifle. However, with some primitive assumptions about the relationship between bore time and muzzle velocity, the period of the displacement-versus-velocity sine wave is easily calculated for a 21" barrel vibrating at 6700 Hz, and a muzzle velocity of 3300 fps, and that period agrees very well with what Vaughn shows.

The sine wave in figure 4-41 of the Vaughn book can be reproduced by plotting a particular trigonometric function. See figure 1. At this point, the analysis accepts without further evaluation, Vaughn's model as representative of the behavior of a typical HV benchrest rifle.



Vaughn explains that, since the velocity of each of the rounds fired to create a group leaves the barrel at a slightly different velocity, it is prudent to adjust the load so as to produce an average muzzle velocity near the maximum or minimum points on the wave. In the case illustrated in figure 1, the best velocities would be just below 3100 fps, and just below 3300 fps. Vaughn further comments that, due to the gravity effect not reflected in the plot, the best velocities are velocities just below the minimum point at around 2900 fps, and just over the maximum point at around 3300 fps. Re-plotting the curve <u>with</u> the effect of gravity included should show the optimum points, as will be done shortly.

As the mission of this analysis is to quantify the effect of tuning, it is helpful to obtain a mathematical expression for Vaughn's sine wave. The equation for a simple sine wave like the one in the figure is:

$$Y_1 = A + B \sin (C + DV)$$

Here, Y_1 is the location of the point of impact due to vibration in inches, V is muzzle velocity in fps, and A, B, C and D are constants used to fit the sine wave to the data. For a sine wave identical to Vaughn's in his figure 4-41, A = 0, B = 0.3, C = -50.1 and D is, in fact, 2 times π divided by the period of the wave, which is 400 fps (not seconds – this is not a sine wave that is a function of time, but one that is a function of velocity). Constant B is the amplitude of the sine wave – in the case of Vaughn's figure 4-41, the maximum value is 0.3 inches above zero and the minimum is 0.3 inches below. Constant C positions the sine wave relative to the values on the horizontal (velocity) axis. Constant A , zero in this case, will be used later to reflect elevation adjustment setting on the riflescope that relates point of aim to mean point of impact.

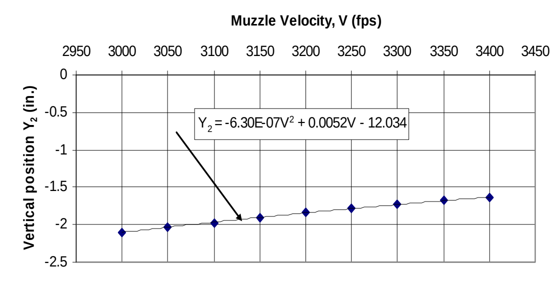
Re-introducing the Gravity Drop

Multiple means exist for calculating the effect of gravity drop for a typical 6mm match bullet at 100 yards. The means chosen was the Sierra “Infinity” ballistics program. The chart in figure 2 was plotted using Microsoft Excel using the Infinity results for a ballistic coefficient of 0.275, standard conditions and for velocities in the range of interest. Excel will fit a trend line to any data on a graph and will display the equation for such a trend line. Since a quadratic most closely fits the physics of gravity acting on a body launched horizontally, such an equation was chosen for the trend line. The form of the quadratic is:

$$Y_2 = E\,V^2 + F\,V + G$$

Here Y_2 is the position of the point of impact, or drop (a negative value) of the bullet due only to gravity in air at standard conditions. And V is, again, muzzle velocity and E, F and G are constants. Specifically, the constants are valued as: $E = -6.30 \times 10^{-7}$, $F = 0.0052$ and $G = -12.034$. The difference between the line and the Infinity points is at worst 0.006 inches.

Figure 2. Gravity effect: Sierra "drop" data for BC=.275 std day



Adding the deflections together results in an expression for the point of impact as a function of muzzle velocity that considers both gravity drop and barrel vibration per Vaughn’s sine wave:

$$Y = Y_1 + Y_2,$$

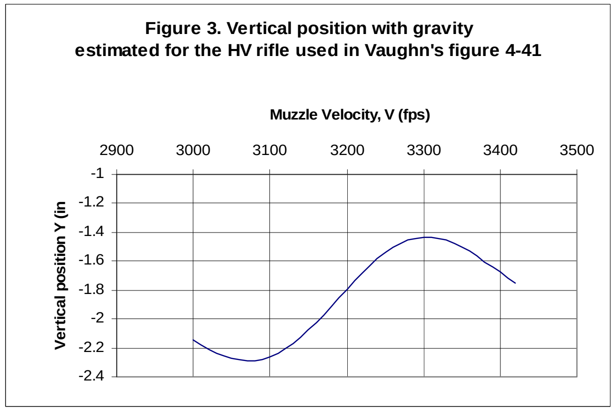
or,

$$Y = A + B\sin(C + DV) + E\,V^2 + F\,V + G$$

where Y is the resulting position, V is muzzle velocity and constants A through G have been determined. Figure 3 is the plot of the combined effects.

The points where the slope of the curve in figure 3 is zero – the “sweet spots” – can be rigorously determined by finding the first derivative of the equation for Y, dY/dV, and setting it to zero. Similarly, by finding the second derivative, d^2Y/dV^2, and setting it to zero, one can find the points where the slope is steepest, or the “sour spots.” This was done, but simple inspection of the curve yields a result with sufficient precision for the purpose of determining the value of tune.

Monte Carlo Simulation Determines the Group Height Component Produced by Velocity Tuning



The equation for Y shown above will allow calculation of the point of impact insofar as it is affected only by gravity and aerodynamic drag (Infinity results) and barrel vibration (Vaughn's sine wave). The myriad of other factors of interest to riflemen perturb individual results, of course. But the objective here remains to determine the effect of velocity tune insofar as it affects displacement due to barrel vibration.

For these purposes, a 5 shot group is produced by firing 5 carefully loaded rounds into the record bull of a target at 100 yards. Any 5 rounds will exit the barrel at 5 slightly different velocities. It is reasonable to assume a normal, or Gaussian, distribution for velocities. As such, velocities tend to be grouped around the average, rather than be uniformly spread over a range. When measuring velocities with a chronograph, it is more likely to fire a round within, say, 10 fps of the average than, say, between 10 and 20 fps of the average. It is more unlikely to see a velocity between 20 and 30 fps, and so on. Vaughn and others suggest that this distribution of velocities contributes to vertical dispersion of holes in the target because of the phenomenon shown in figure 3. If we were to know 5 typical velocities with an average of 3305 fps, we could calculate Y for each, and calculate the range of Y values to get the group height contribution at a "sweet spot" as determined by barrel vibration, and similarly, with 5 typical velocities around 3190 fps, one could compute group height contribution at the "sour spot." The difference in vertical height contributions would be the value of velocity tuning.

Since the objective is to obtain a quantitative value for the effect of velocity tuning, some rigor is warranted in the computation. Excel has a feature that will generate any number of velocities with a specified average and a specified measure of tendency to vary about that average. This is done by combining a random number generator with an equation for the normal (Gaussian) bell-shaped curve. This is much easier that it sounds. One need only place the expression

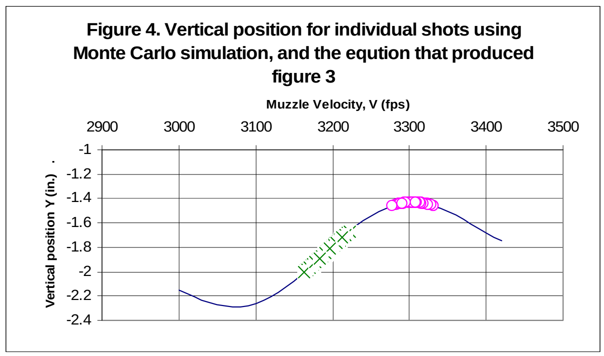
=NORMINV(RAND(),M,S)

in a cell; where M is the desired average velocity and S is the desired standard deviation – the measure of the tendency of velocities to be near the average. Placing this expression in 5 Excel cells produces 5 random velocities from a population that has an average velocity of M and a standard deviation of S. If the reader is going to build a spread sheet with this feature, turn off the "auto-calculation" feature first. When this is done, a new set of random numbers will be produced each time the F9 key is pressed, rather than each time a change is made to the worksheet.

From the 5 velocities, the equation for Y above can be used to determine the point of impact for each shot, and by subtracting minimum from maximum in the set of 5 points, (use =MAX(y1, …, y5) and =MIN(y1, …, y5)) the Y range as affected by vibration is calculated for that specific group. Repeating the calculation many times yields average values for the phenomenon.

The Excel Worksheet.

An Excel Worksheet was created that models the Vaughn sine wave, gravity drop and velocity statistics as described above. The average contribution to group height is calculated for 100 groups with a velocity at the "sweet spot" and an average for 100 groups with a "sour spot" velocity. The heights are then subtracted. The effect of standard deviation and of choice average velocity can be explored by changing any of the 3 parameters – sweet spot velocity, sour spot velocity or standard deviation. A result of the calculations appears in figure 4. Here the X symbols represent a large number of shots fired at the "sour" load, and the O symbols represent the same number of shots fired with a "sweet" load. See the relatively large difference in vertical position (Y) among the X symbols as compared with the small difference among the O symbols.



Results of the Investigation into Vaughn's Sine Wave

For the Vaughn sine wave in his figure 4-41, a "sweet spot" exist at 3305 fps, and a "sour spot" exists at 3190 fps. This is clear once gravity drop is reintroduced, viz. figure 4. If one assembles loads with a standard deviation of 12 fps, the average vertical heights corresponding to the two average velocities are 0.0138 inches and 0.1581 inches; the difference being 0.1443 inches. Each run of the spreadsheet yields a slightly different result, but these values are typical. This means, accepting the Vaughn model as representative of varmint-class benchrest rifles and 12 fps as a typical standard deviation, that a worst-possible tune will cost one something on the order of 0.144 inches in the vibration component of vertical group size – well worth fixing but not something that would explain a rifle that had been shooting mid-twos suddenly shooting fives. That's if the tune is absolutely in the worst possible place – possible, but not likely.

The effect of velocity standard deviation can also be explored with the tool. Looking at the model of Vaughn's rifle, the simulation predicts that "sour" Y range doubles in size and "sweet" Y range increases about 4 times. The difference, or the effect of tuning, doubles as well. The "sweet" Y ranges are small, indeed, hence the increase in "sour" Y range and difference just about matches. This means that if tune is good, ammunition producing large standard deviations doesn't hurt much at 100 yards. A "sour" tune demands low standard deviation ammunition.

Applying the Method to Another Rifle

Accepting Vaughn's method and his figure 4-41 data as representative of varmint class benchrest rifles in general, tests were performed to see where the range of loads used in a Light Varmint rifle would fall relative to the sine wave created for Vaughn's rifle. The test was not performed to reproduce Vaughn's results, but to see if the range fell closer to the "sweet spot" or the "sour spot," for a particular rifle of slightly different configuration.

The rifle used for this phase was built by Bruce LaChapelle using a drop port Viper action, Krieger 14" twist barrel, Borden Rimrock stock and a Leupold 36X scope. Loads were assembled using Euber "zero-die" 66 grain bullets seated to touch the rifling, VV N-133 dispensed from a Harrell powder measure, and Federal 205M primers. Case necks were turned to 0.0086 for use in the 0.262" neck chamber.

Table 1 – Vertical Position of Groups Fired from Viper DP LV Rifle

Harrell Measure Setting	Measured at Range (822 ft elev.)		Calculated Drop @		Calculated Drop @ Standard Conditions	Y Position Corr. to Standard Conditions
	Velocity Corr. to muzzle	Y Position	822 ft elev.	**Y minus drop**		
52	**3227**	-0.939	-1.798	**0.859**	-1.810	-0.950
52.5	**3249**	-0.768	-1.773	**1.005**	-1.785	-0.780
52.5	**3273**	-0.784	-1.746	**0.963**	-1.759	-0.796
53	**3300**	-0.821	-1.717	**0.897**	-1.730	-0.833
53.5	**3306**	-0.730	-1.711	**0.981**	-1.724	-0.742
54	**3353**	-0.803	-1.663	**0.860**	-1.676	-0.816
		see note		see note		see note
Note: Y position is measured from an arbitrary horizontal line drawn on the target						

A brief test conducted at the Pine Tree Rifle Club 100 yard range yielded measured velocities and vertical locations of median bullet holes. Chronograph measurements were corrected for the distance from the muzzle, and vertical locations were corrected for the gravity drop predicted for the prevailing conditions using the Infinity program. The ambient temperature was 59ºF, so no temperature correction was required. The test results appear in the accompanying table.

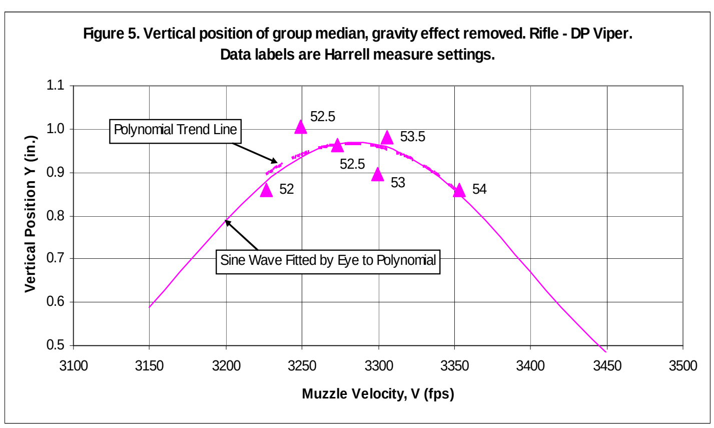


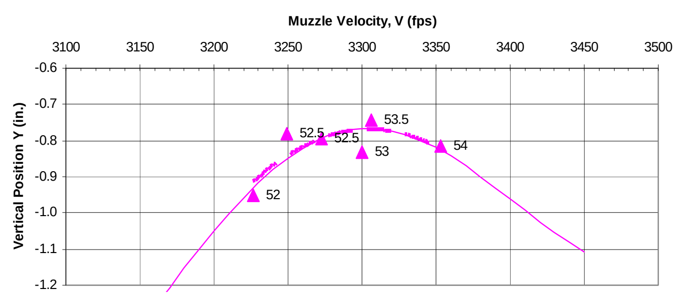
Figure 5. Vertical position of group median, gravity effect removed. Rifle - DP Viper. Data labels are Harrell measure settings.

A word of caution is appropriate here. Do not attempt to reproduce the results shown here without establishing the safety of loads by using sound reloading practices found in published manuals. If in doubt, consult your gunsmith. The loads appearing here were used safely in the author's rifle, following the author's reloading procedures, and at the stated test conditions. Powder measures and operating techniques vary, so the same setting can produce significantly different, and potentially excessive, charges. No representation is made by the author or the publisher regarding the safety of these loads in other rifles or at other conditions.

At this point, there is no correction for point of aim or location of the horizontal datum on the target. Use of the constant A in the equations cited above will take care of this.

As Vaughn did, a graph was made of position on the target with respect to actual velocity, first, with the effect of drop removed ("Y minus drop" column in the table). The points in figure 5 (solid triangles) result. It is not possible to plot a sine wave with any credibility from only these 6 points. An alternative is to assume a sine wave similar to Vaughn's would tend to fit these points. Proceeding thus, Vaughn's model is neither validated nor questioned, but insight is gained into the behavior of other rifles. First, to see if the new Y points are on the "sour" sloped part of a curve or on the "sweet" flat maximums or minimums, a trend line (an Excel feature) was fit to the plotted points. The trend line appears also in figure 5 as a bold, dashed line. The trend line is a second-order polynomial. It is reassuring that: a) the points are not centered on steeply-sloping sides of the polynomial, and b) the maximum value is near the center of the data, and is at a similar velocity to that of a maximum in Vaughn's sine wave.

Figure 6. Vertical position of groups, including both vibration and gravity for the DP Viper test.



The third step is to fit a sine wave similar to Vaughn's to the regression line. Constant A was adjusted to account for the location of the arbitrary datum. Constant B was held to the same value as Vaughn used: 0.3. Constant D was adjusted to fit the width of the plotted polynomial that fit the test data, and constant C was increased slightly to align the maximum of the sine wave with the maximum of the polynomial trend line. The resulting sine wave is also plotted in figure 5; it's the fine, solid line.

The final step is to reintroduce the effects of drag and gravity for the standard conditions. The same process was used here as was used in the study of the Vaughn sine wave, and the results of this step appear in figure 6. Note the indicated "sweet spot" is around 3310 fps, and the test points were not over a wide enough range to be sure of capturing the "sour spot."

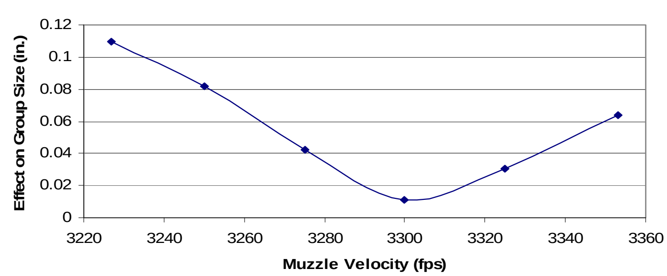
The plotted points are labeled with the associated Harrell measure setting. Note the irregular relationship between velocity and powder measure setting, and also that two identical settings (52.5) produced different velocities. Causes could include cooling or warming of the powder over the course of the session, and factors such as humidity that may affect the density of the powder, as ambient air enters the powder reservoir during the course of the test. These factors would have to be considered when attempting to load to a particular muzzle velocity under match conditions, but should not be a factor in predicting point of impact with a known muzzle velocity. I think.

New Questions

Given the model of the effect of velocity tuning that appears in figure 6, what would a Monte Carlo simulation say was the effect of producing average velocities at various points between 3227 fps on the low end and 3353 fps on the high? These were the high and low velocities of the Viper tests.

Figure 7 resulted from exercising the model, "firing" 200 groups – 1000 "rounds" at each of the 6 velocities identified with a plotting symbol. Even within this range, choosing, or settling for, the lowest average velocity would result in groups that tend to be about a tenth of an inch larger than groups loaded to the "sweet spot." Shooting with a 70 fps too low a muzzle velocity produces an effect that would drop a winning aggregate to a mediocre one. These results are for the same velocity standard deviation as used earlier in the study – 12 fps.

Figure 7. Monte Carlo analysis shows contribution of velocity tune to group size - DP Viper @ 100 yds per the behavior model in Figure 6.



And see the relative slope of the line above and below the "sweet spot." Ever had one of the masters of the sport say when things go sour, you need to increase (not decrease) the load? This advice seems to make sense if the rifle is operated near a "peak" of the sine wave (figure 6) rather than in a "valley." If the models shown in the preceding figures, again, are representative, a "valley" load would not be commonplace, so, within reason, increasing is the right direction (of course, don't exceed maximum safe loads).

Some Conclusions

Accepting the Vaughn model as generally representative of the performance of varmint-class benchrest rifles, a combination of elevated velocity standard deviation and a load producing "sour spot" velocities, nearly 0.3 inches can be added to a vibration term in a general expression for bullet position. Vaughn suggests factors add geometrically, not linearly, and one should consider evaluating the square-root of the sum of squares of independent variables.

Vaughn correctly cautions that independent factors contributing to group size (even to vertical dimension) do not combine linearly. Even knowing the "sweet" to "sour" contribution attributable to a velocity standard deviation of 12 fps is 0.144 inches, it is probably wrong to conclude on that groups would be 0.144 inches higher. If the contributions of all other factors affecting group height were known, it would then be possible to combine all factors and produce a credible number for the effect of tune on overall group height. That said, if the vibration effect is large by comparison, what is true for the vibration effect is true for the group height.

Another observation is, if a load is tuned to the "sweet spot" a somewhat elevated standard deviation will have a very small effect. By this I mean going from a standard deviation of 12 to one of 24. This is not true of the "sour spot."

If the grouping problem one seeks to correct is in a rifle formerly shooting in the low 2's and high 1's, that suddenly shoots half inch groups (all with velocity standard deviation measured as 12 or so fps), the problem isn't likely to be correctable by tuning. If 3's start appearing regularly, tuning may make 2's reappear in their place. See "Future Work."

It is reasonable to expect other (maybe not all other) varmint class benchrest rifles of conventional configuration to behave like the rifle used by Vaughn. This is drawn on the somewhat questionable basis of the test of only one other rifle.

According to the model, the "sweet spot" is about 40 fps wide, not to put too fine a point on it. The corresponding charge of VV N-133 is about one unit (like the difference between 52.5 and 53.5) on the Harrell measure, or about half a grain of powder. The difference between a "sweet" and "sour" spot is about 115 fps. That number is for the Vaughn rifle. The difference would be slightly larger for the rifle I tested. The corresponding adjustment on a Harrell measure would be just under 2 units, like the difference between 52 and 54, which is about one grain of powder. This does not agree with what some have observed.

Some Vihta Vuori published information would suggest a 45ºF change in powder temperature would require half a grain change in charge (6 ppc), or a one unit on the Harrell measure, to maintain the same muzzle velocity. It would follow that half that temperature would put the load outside the 40 fps "sweet spot" if one started from the center. Implications to pre-loading are interesting to ponder.

Future work

It would be valuable to pursue combining the vibration effect with other significant factors affecting group size. This could be done with an expansion of the Monte Carlo method, but requires a model of the other factors.

Vaughn's model is not the only one that would lend itself to the procedure described above. Others have used finite element models of the rifle system to determine point of impact insofar as it is affected by the motion and position of the muzzle plane at the instant of bullet release. This FEM analysis determines the transient response of the system, and should produce precise and accurate results. But using such a model's results in an analysis similar to this one requires an accurate way of calculating bore time from muzzle velocity. This is currently lacking, and is likely load-specific.

In the short term, I'd like to do two things. First, using the Viper, explore more of the sine wave – over a full period – with more data points; this would reproduce Vaughn's results. Second, explore same with a different barrel; one, say, a tad longer or shorter – and maybe with a sporter or a rail gun.

Step-by-Step Instructions for Performing a Tuning Test:

Purpose of step-by-step instructions. I have been told that a faster path to mastering benchrest shooting is to dismiss all the technology as some sort of magic, and spend time shooting groups in the wind. The truth of this argument has been adequately and repeatedly demonstrated. However, the pleasure I derive from this game is in attempting to understand the physics. The editor's boss tells me that one or two of the readers of this magazine may be similarly motivated, and may be interested in duplicating the above study with another rifle. He suggested I provide a step-by-step instruction for the process. More seriously - I believe something could be learned from multiple investigations, and would be delighted to see the results of any and all tuning tests.

Assumed abilities, familiarity with Excel and a ballistics program. The following instructions assume the experimenter has a basic familiarity with the Microsoft Excel software, or has a friend, or family member with said familiarity with whom to collaborate. One need not write "macros." It is further assumed that a ballistic program such as Sierra's "Infinity" or the equations in Mr. Pejsa's book, "Modern Practical Ballistics." (Kenwood Publishing, Minneapolis, 1989) is available. Finally, the experimenter or an associate has the ability to shoot a reasonable approximation of "one hole" 3-shot groups at 100 yards.

Construction of target. Prepare the targets by drawing a horizontal line a consistent distance from the aiming point to be used – something like a line across the paper tangent to the bottom of the aiming square on a regulation group target.

Shooting process. Record the elevation above sea level of the range, ambient temperature and distance from the muzzle to the center of the gage section of the chronograph. Select one bullet type of known ballistic coefficient with which suitable precision can be expected. Record the ballistic coefficient. Select a variety of safe charges, and, with a single relative point of aim and scope adjustment, fire several 100 yard, 3-shot groups, each with a different charge. For each group, record the charge, average velocity as measured with the chronograph, and the vertical distance from the horizontal line (datum) to the center of the group.

Processing of data. Correcting ballistic coefficient for range conditions - This can be done in the Excel spread sheet by placing the function for corrected BC (CBC)

= (BC) * ((459.4 + temp) / (518.4-elev /280)) * EXP(elev / 31654)

in one of the cells. I place the published ballistic coefficient (BC), range temperature (temp) and elevation above sea level (elev) in cells and have the function read from these cells. In Excel, EXP is the mathematical function that takes the constant "e" and raises it to the power of what appears within the parentheses. This equation is taken from chapter 13 of Pejsa's book, barring yet another senior moment on my part. I will call the ballistic coefficient adjusted for site conditions CBC, or corrected ballistic coefficient. .

Calculating actual average muzzle velocity, V, for each group – With the CBC, and the distance from the muzzle to the center of the chronograph "gage section," the average measured velocities for the groups and the equation,

$$V = ((\text{meas. vel.})^{.5} + (\text{dist} / (332 * \text{CBC})))^2$$

Here, V is actual muzzle velocity in fps, "meas. vel." is what is read off the chronograph, "dist" is the distance in feet from the muzzle to the chronograph, and CBC is the corrected ballistic coefficient. Alternatively, V can be determined by some trial-and-error with a ballistics program for the published BC, distance, measured velocities and ambient conditions.

Correcting for gravity - I used Sierra's Infinity program. Simply input the site conditions, published ballistic coefficient and each average V into the program and note the "drop" at 100 yards. Adjust the measured position of the group centers by subtracting the drop due to gravity. Pay attention to the plusses and minuses. You should wind up with a situation where a high "Y minus drop" corresponds to a high position on the target paper. At this point, enough information has been gathered to construct the first 5 columns of "table 1" above.

Calculating drop for standard conditions - This is done exactly the same way as was done for test conditions, except that the default ambient conditions were used in the ballistic program. That takes care of column 6. Column 7 corrects "Y minus drop" to produce total drop for standard conditions. The table is now complete.

Plotting and curve fitting - If you or your Excel assistant has done lots of plotting, just have at it. Otherwise, here is a step-by-step of how to exercise Excel.

1. Highlight the corrected velocity column and the "Y minus drop" column.
2. Click on the graph icon.
3. Select "XY (scatter)" type, and the dots-only sub-type on the menu that appears.
4. You may choose to add titles and otherwise manipulate the appearance of the chart at this step. When done,
5. Right-Click on any one of the points plotted. On the new menu, select "add trendline." Select a polynomial of order 2.

6. You may want, at this time, to go into trendline “options” and have the equation of the trendline displayed on the chart. This should have produced something similar to figure 5, and repeating the process for the columns for velocity and the last column should produce something like figure 6.

Interpretation of data. If the curves you have produced look sort-of, kind-of, approximately like they pass through the data, good. This should be the case if the velocities are within a range of, oh, 150 fps. If your data covers more like 300 fps, you may have to try a 3^{rd} order polynomial. But let’s assume the 150 fps case, and proceed. If your “figure 5” looks like mine, your “sweet spot” is where your “figure 6” reaches its maximum. If your “figure 5” curves up at both ends – “similes” rather than “frowns” – your “sweet spot” should be where your “figure 6” curve reaches minimum. If your “figure 5” just tends up or down, you may have captured the “sour spot” and you may wish to explore a velocity range away from where you have been.

References:

Pejsa, Arthur J., “Modern Practical Ballistics.” (Kenwood Publishing, Minneapolis, 1989)

“Infinity” Exterior Ballistics Software, Version 2.2. Sierra Bullets, LLC. 2001.

Vaughn, Harold R., “Rifle Accuracy Facts.” Precision Shooting, Inc., Manchester, CT. 1998.

VihtaVuori Reloading Data DTD 1-99